

# Nonlinear Dynamical Approaches to Human Movement

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Nonlinear dynamics and dynamical systems approaches and methodologies are increasingly being implemented into biomechanics and human movement research. Based on the early insights of Nicolai Bernstein (1967), a significantly different outlook on the movement control “problem” over the last few decades has emerged. From a focus on relatively simple movements has arisen a research focus with the primary goal to study movement in context, allowing the complexity of patterns to emerge. The approach taken is that the control of multiple degrees-of-freedom systems is not necessarily more difficult or complex than that of systems only comprising a few degrees of freedom. Complex patterns and dynamics might not require complex control structures. In this paper we present a tutorial overview of the mathematical underpinnings of nonlinear dynamics and some of its basic analysis tools. This should provide the reader with a basic level of understanding about the mathematical principles and concepts underlying pattern stability and change. This will be followed by an overview of dynamical systems approaches in the study of human movement. Finally, we discuss recent progress in the application of nonlinear dynamical techniques to the study of human locomotion, with particular focus on relative phase techniques for the assessment of coordination.

*Key Words:* \\Au, need 3 or 4, but NOT from title\\

## Dynamical Approaches to Movement Coordination

### *Exploiting Dynamics*

One of the most prominent issues in the study of movement coordination and control is how humans or animals manage to orchestrate individual units into coherent patterns of coordination. This question, originally formulated early in the last century by the Russian movement scientist Nicolai Bernstein (e.g., Bernstein, 1967), is widely known as the *degrees of freedom problem* of movement coordination and

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control. Even in what we perceive as the most basic patterns of coordination, such as locomotion and postural control, these issues are of critical importance. Bernstein approached the question of controllability of all our motor units, muscles, and joints from an integrated perspective in which biomechanical, neural, and physiological constraints interact. The key research questions raised by Bernstein inspired new avenues of research into how people learn complex patterns of coordination (e.g., Latash, 1998; Newell et al., 1989). The question as to how individuals learn to integrate and coordinate the multiple degrees of freedom is not only important for analysis of elite performance such as that of athletes or skilled musicians, but also it will help us better understand coordination problems in individuals with movement disabilities.

According to Bernstein, coordination is not defined as the action of single elements but instead reflects the common action of these separate elements (Bernstein, 1967, 1996). More specifically, Bernstein defined coordination as overcoming excessive degrees of freedom, thereby turning joints, muscles, and motor units into controllable systems. In the Bernstein perspective on motor learning, these changes in the control over the different degrees of freedom are clearly reflected in the different stages of skill acquisition. In the first stage some degrees of freedom are frozen to simplify the control problem. In the second stage a gradual release and incorporation of degrees of freedom into the movement task is observed. Finally, in the last phase of learning the learner is able to exploit nonmuscular forces (such as intersegmental dynamics, gravity, etc.) or other reactive phenomena to further refine the execution of the task (Bernstein, 1967). Refinement of movement, or the mastery of coordination, is accomplished by conveying the necessary impulses at the required moment.

Evidence of these three stages has been observed in a variety of complex movement tasks, such as handwriting and skiing (e.g., **Newell & Van Emmerik, 1987**; Van Emmerik, 1992; Vereijken et al., 1992; 1997). More current notions focus around the universality of the principle of freezing and releasing degrees of freedom during the learning process (Broderick & Newell, 2000; Latash, 1996). Bernstein's formulation of the final phase of learning, namely the exploitation of reactive phenomena, has also spawned much debate in the biomechanics and motor control literature. These debates focus around the separability of active and reactive forces and the degree to which certain aspects of movement, such as the swing phase in locomotion, are indeed passive (e.g., Van Ingen Schenau & Van Soest, 1996; Whittelsey et al., 2000).

A fundamental guiding principle espoused in this paper is that the existence of many degrees of freedom is not a curse but instead facilitates control, although it may complicate initial learning. On the contrary, multiple or redundant degrees of freedom allow for the selection of smaller subsets of these degrees of freedom into efficient, adaptable task-dependent coordinative structures (Greene, 1982; Kugler & Turvey, 1987; Turvey, 1990). In addition, the existence of numerous degrees of freedom allows the organism to approximate a wider variety of desired behaviors, again simplifying the control problem. Our view of control follows the distinction made by Greene (1982) and Turvey (1990) between executive ballpark activation and subsystem tuning. To achieve a certain movement pattern, the controller might activate subsystems without knowing the actual overall outcome, as the tuning of the subsystems will play a role in this as well (Turvey, 1990). Greene (1982) has argued that the reward for this initial ballpark "ineptness" is eventual versatility.

## *Synergy Formation in Human Movement*

In a recently discovered publication by Bernstein (1996), four levels of the construction of movement are identified. The first level is that of *muscle tone*, focusing primarily on postural muscles and the maintenance of body orientation. The second level, *muscular-articular links or synergies*, subserves all possible patterns of locomotion in the terrestrial environment. Higher levels in the hierarchy are defined at the level of *space* and *actions*. Compared to the level of synergies, these two higher levels incorporate aspects of perception and intention in action.

According to Turvey and Carello (1996), dynamics and fundamental principles of synergies are revealed most elegantly through the study and modeling of oscillatory movements. A major impetus for the development of such models was provided through the work of the physiologist Von Holst (1939/1973). Von Holst distinguished between two different forms of coordination, termed absolute and relative coordination. Two oscillators are in *absolute* coordination when they are phase locked. In contrast, oscillators in *relative* coordination maintain tendencies to particular phase couplings but exhibit a much wider array of observed frequency and phase relationships.

Modeling approaches at the level of synergies, inspired by the work of Von Holst, have focused on the level of observables without making assumptions about the internal details of the component oscillators. Specifically, relative phase has been advocated as a variable through which basic principles of organization at the level of synergies can be accomplished. This approach is represented by proponents of the synergetics (e.g., Haken, 1977; Haken et al., 1985; Kelso, 1995) and the natural physical approaches to movement coordination (Kugler & Turvey, 1987; Turvey, 1990; Turvey & Carello, 1996).

In the synergetics approach a distinction is made between collective or “order” parameters and “control” parameters. *Order* parameters are state variables that identify the macroscopic aspects of the system and identify the collective behavior of the components involved. The stability and transition dynamics of a synergy can be revealed by the systematic manipulation of a nonspecific *control* parameter. Such control parameters can be used to reveal regions of stability and instability in coordination. Phase relations between body segments are considered order parameters because of their fundamental reflection of cooperation between components in the system. In later sections we will discuss the applicability of relative phase techniques to the identification of stable coordination modes as well as transitions between these modes in human movement.

These dynamical models of human movement are consistent with developments in biology and medicine, in which biological structure and function is recognized as a complex spatiotemporal organization. Different diseases emerge when the normal organization breaks down and is replaced by some abnormal dynamic. In the dynamical diseases approach, the nature of abnormal biological rhythms is assessed on the basis of nonlinear mathematical models in which control parameters are essential in inducing a pattern change (Glass, 2001; Glass & Mackey, 1987; Winfree, 1980).

A hallmark of these dynamical systems models is that the existence of multiple rhythms or periodicities does not have to imply that the underlying organization of the movement control variables has to be complex, i.e., that all these periodicities have to be represented in the control system. On the contrary, ma-

nipulation of only one control parameter can generate different types of dynamics, ranging from no observable periodicity to simple and complex oscillations.

Over the last three decades, dynamical systems approaches have made significant contributions to our understanding of stability and change in biological systems. This research has countered the general assumption that pathologies are universally characterized by increased variability and instead argue for a functional role of variability in healthy systems. Lack of variability from this perspective is characteristic of dysfunction in system performance, frailty and disease (Lipsitz, 2002).

## Nonlinear Dynamics: Tools to Assess Stability and Change

We now turn to a tutorial overview of the basic tools of nonlinear systems approaches for the assessment of pattern stability and change. We begin with the analysis of discrete movement toward a single equilibrium point and later show the connections with rhythmic movement as discussed above.

### *Fixed Points and Stability*

Consider the mass-spring-dashpot system depicted in Figure 1a and modeled by the following linear differential equation:

$$m\ddot{x} + b\dot{x} + k(x - x_{eq}) = 0, \text{ where } m, b, k > 0. \quad (1)$$

In Equ. 1,  $m$  represents the mass,  $b$  the damping coefficient, and  $k$  the stiffness;  $x$ ,  $\dot{x}$ ,  $\ddot{x}$  denote position, velocity and acceleration, respectively. We already know the usual behavior of this system given our intuition about the mechanical components. The mass will typically bounce back and forth and the damper will dissipate energy until the mass comes to rest at the equilibrium position of the spring ( $x_{eq}$ ). This sort of intuition often serves us well although we can run into trouble with more complex systems, especially ones with nonlinear components. For example, consider a comparable system with nonlinear stiffness in the spring:

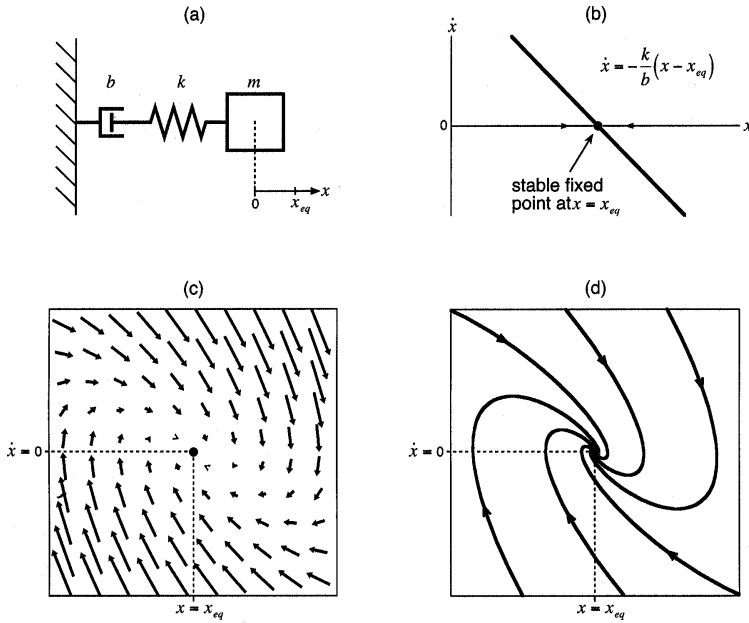
$$m\ddot{x} + b\dot{x} + k \sin(x - x_{eq}) = 0. \quad (2)$$

With the mass close enough to the equilibrium position, this new system behaves nearly the same as the linear case since  $\theta \approx \sin \theta$  for small  $\theta$ . But what if the mass is not close enough to equilibrium? The sine term in Equ. 2 introduces a nonlinearity, whereby the spring's restoring force is no longer proportional to the deviation from equilibrium. In fact, for certain values of  $x - x_{eq}$ , this "restoring" force pushes the mass further from equilibrium! But before going very far with this example, we first lay the technical groundwork for a more detailed discussion of nonlinear dynamics in the neuromuscular system.

Assume for the moment that the mass in Figure 1a is already in motion and its inertia is small relative to the damping. Under these conditions, i.e., when  $m\ddot{x} \ll b\dot{x}$  the linear system is overdamped and we can ignore the contribution of the inertial term in Equ. 1. The equation of motion then becomes:

$$b\dot{x} + k(x - x_{eq}) = 0. \quad (3)$$

This overdamped scenario may seem contrived but it makes a useful starting point for at least two reasons. First, it paves the way for an explanation of an oscillator



**Figure 1 — (a) Mass-spring-dashpot system with mass  $m$ , linear spring constant  $k$ , and damping coefficient  $b$ . (b) One-dimensional vector field diagram for the overdamped case. (c) Two-dimensional vector field diagram for the case without the overdamped assumption. (d) Corresponding two-dimensional phase portrait with several representative trajectories that converge on the stable fixed point.**

model discussed later. Second, the overdamped case simplifies our presentation of some key concepts. From Newton’s second law we know that  $F = ma$  and so the position of an inertial system—like the mass in Figure 1a—is normally described by a second-order differential equation that relates force and acceleration. With inertia eliminated from Equ. 1, the result is a much simpler first-order system where  $\dot{x}$  is the highest time derivative of position.

The solution to Equ. 3 is a family of functions parameterized by the initial position of the mass,  $x_0$ , in addition to the equilibrium position and the damping and spring coefficients:

$$x(t) = x_0 e^{-\frac{k}{b}t} + x_{eq} (1 - e^{-\frac{k}{b}t}). \quad (4)$$

Now that we have the position of the mass expressed as a function of time, the long-term behavior of the system becomes apparent. In particular, as  $t \rightarrow \infty$ , the exponential terms go to zero and so  $x(t) \rightarrow x_{eq}$  as expected from a static analysis where  $k(x - x_{eq}) = 0$ . Equation 4 not only confirms our intuition about the equilibrium point but also tells us something about the transient behavior, which is useful because time never goes to infinity in practice. More specifically, if we think of  $x_0$  as a disturbance from the equilibrium, then Equ. 4 tells us the system is stable. That is, the system dissipates disturbances at an exponential rate quantified by the characteristic exponent,  $^{-k/b}$ . (Alternatively, one might express the exponential rate

by the time constant  $\tau = 1/|b/k|$ .) When the characteristic exponent is positive, for instance when  $b < 0$  and the “dashpot” injects energy, then the system is unstable and the disturbance instead grows at an exponential rate. Although the simplified analysis so far has revealed closed-form solutions, the problem of solving differential equations can become infeasible with more complicated nonlinear systems and with systems where we have empirical data but no equations of motion. Somewhere between closed-form techniques and intuition are the approaches based on graphical constructs such as phase portraits and vector field diagrams.

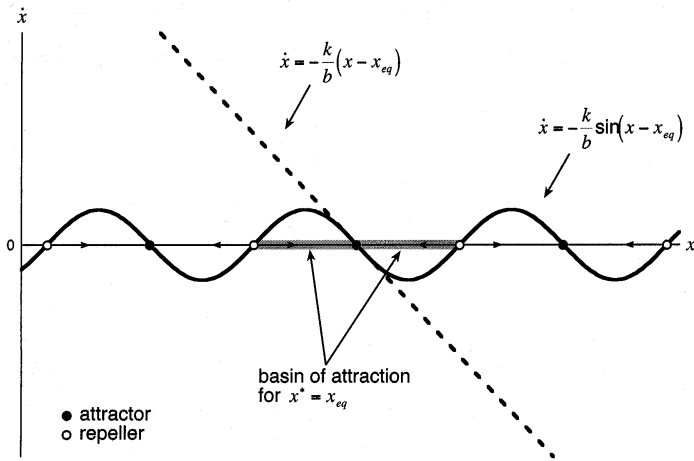
### *Phase Portraits and Vector Field Diagrams*

A vector field diagram depicts the velocity of an imaginary particle traveling through an abstract state space. For the overdamped case, state space and physical space happen to coincide, although more generally state space may involve quantities such as velocity, temperature, voltage, neurotransmitter concentration, and so on. Figure 1b shows the vector field diagram associated with Equ. 3. In one dimension, the diagram is simply a plot of velocity ( $\dot{x}$ ) vs. position ( $x$ ) with arrows indicating the direction of movement. In two dimensions, such plots use arrows to show the velocity at various points in the state space, and therefore to represent the flow of the dynamical system. (Imagine a spray of ink droplets in running water; the flow will stretch the droplets to match the local speed and direction of the water.) Figure 1c shows a sample two-dimensional vector field diagram, derived from the original mass-spring-dashpot system without the overdamped assumption.

Phase portraits, on the other hand, show representative trajectories through the flow field. They illustrate the path followed by an imaginary particle once released into the state space. The arrows along the horizontal axis in Figure 1b show a simple kind of phase portrait, but more typical is a two-dimensional portrait as in Figure 1d. Phase portraits are often easier to interpret than vector field diagrams, although the drawback is that they hide information about time (i.e., the particle’s velocity). As long as the graphics remain uncluttered, we sometimes get the best of both worlds by combining phase portraits and vector field diagrams onto one plot.

Returning to the overdamped, one-dimensional system, the vector field diagram reveals a key point in state space. The point where the curve in Figure 1b intersects the horizontal axis is special because velocity is zero and the flow stops. Such points in state space are commonly called fixed points and their presence often dictates the qualitative behavior of a dynamical system. Even for complex nonlinear systems that we cannot solve in closed form without consulting a good reference (e.g., Boyce & DiPrima, 1986), or that we can’t solve at all, we can still perform a local analysis at the fixed points. Thus the discovery of fixed points can lead to both quantitative and qualitative information about a dynamical system.

For linear, one-dimensional systems, analysis of a fixed point is simply a matter of examining the slope of the line in the vector field diagram. This slope gives the aforementioned characteristic exponent, with stable and unstable fixed points distinguished by negative and positive slopes, respectively. With stable fixed points, for instance, rightward disturbances from equilibrium are opposed by a leftward flow since velocity is negative to the right of the fixed point. Leftward disturbances are similarly opposed by a rightward flow that restores equilibrium at



**Figure 2** — Vector field diagrams for the overdamped mass-spring-dashpot system with both linear and nonlinear stiffness in the spring (dashed and solid curves, respectively). Closed circles denote stable fixed points, i.e., attractors. Open circles mark the unstable fixed points, i.e., repellers. The attractor at  $x^* = x_{eq}$  has a basin of attraction (gray region) that extends to its neighboring repellers, which in turn separate the basins of attraction for the stable fixed points.

a stable fixed point. In higher dimensions, the concept of slope must be generalized to the Jacobian matrix along with its eigenvalues and eigenvectors. Many of the basic principles are the same whether we deal with slopes or Jacobians (see Strogatz, 1994, for a more in-depth treatment).

For the nonlinear case in Eq. 2, we can use the same assumption about an overdamped mass to turn the example into a first-order system:

$$b\dot{x} + k \sin(x - x_{eq}) = 0. \tag{5}$$

This differential equation is solvable in closed form but the result is hardly illuminating. The vector field diagram in Figure 2, however, is much easier to interpret. We see almost immediately that the system has many fixed points,  $x^*$ , some of which are stable and some unstable. The fixed point  $x^* = x_{eq}$ , in particular, is stable given the negative slope at  $x = x_{eq}$ . Moreover, we know the stability of this fixed point is in some sense equivalent to the linear case since the sinusoid is roughly linear at the zero crossings. Fortunately, we can be more precise about this equivalence. Once we restrict our analysis to the neighborhood of a given point—fixed point or not—we can use a Taylor expansion to eliminate the nonlinearity in  $x$ . In general, we need an infinite number of terms in this Taylor expansion, but more typically the first-order expansion with just two terms is sufficient for a stability analysis. Not surprisingly, the first-order term is one and the same as the slope at the given point. For the overdamped nonlinear example, the dynamic in question is:

$$f(x) = -\frac{k}{b} \sin(x - x_{eq}). \tag{6}$$

and the derivative of  $f(x)$  gives the slope that we seek:

$$f'(x) = -\frac{k}{b} \cos(x - x_{eq}). \quad (7)$$

At the fixed point  $x^* = x_{eq}$ , the derivative is  $f'(x_{eq}) = -\frac{k}{b}$  as in the linear case. This analysis may seem circular, but what the Taylor expansion really tells us is that for nonlinear systems we can still use slope to quantify stability of a fixed point, insofar as the higher-order terms, those involving higher derivatives of  $f$  and higher powers of  $x$ , are negligible near  $x^*$  (see Strogatz, 1994, pp. 16–25 for more detail).

Our emphasis on fixed points may seem like a narrow view of dynamics, especially where ongoing or rhythmic phenomena (such as during gait) are of considerable importance. Nonlinear dynamics is a broad subject and choices must be made about which concepts to emphasize. Nevertheless, fixed points do play a role in the analysis of systems with ongoing dynamics, especially ones with periodic behavior. In the next section we highlight the connection between gait and two theoretical constructs: coupled oscillator models and Poincaré maps.

### *Coupled Oscillator Models*

Rhythmic movements, such as walking, running and swimming, are often unconscious, automatic activities. In mammals the basic rhythm for such movements is supplied by networks of spinal neurons called central pattern generators (CPGs). Most evidence for the existence of CPGs comes from case studies of human patients with spinal injuries (e.g., Calancie et al., 1994; Dimitrijevic et al., 1998), as well as from experiments on reduced preparations of lower mammals such as decerebrate cats (Whelan, 1996). The way neurons interact to generate a pattern is an intriguing subject from a nonlinear dynamics perspective. In this section, however, we abstract away from neurons and focus on the way a pair of CPGs interact and coordinate their individual behaviors.

Consider a trivial dynamical system of the form:

$$\dot{x} = v. \quad (8)$$

The state of this system goes to infinity at a constant rate given by the velocity,  $v$ . (The vector field diagram is simply a line parallel to the horizontal axis, with no intersections and therefore with no fixed points.) Now suppose we add a level of interpretation and assume that  $x$  describes the position along a circular path. The path need not be truly circular but rather topologically equivalent to a circle. Put differently, a continuous transformation can be used to change or “morph” the path into a circle and back again. Such transformations are subject matter for topology, a branch of mathematics that Stewart (1989) described as “rubber sheet geometry” or the “mathematics of continuity.” The important characteristic for our present example is that the path wraps around, and by convention it does so every  $2\pi$  radians of travel. Thus if we rewrite Equ. 8 in terms of angular quantities (i.e.,  $\theta = \omega$ ), then the system becomes a simple model of a central pattern generator with oscillatory behavior.

Now suppose we have two oscillatory CPGs with symmetric coupling modeled by a sine function:

$$\begin{aligned} \dot{\theta}_1 &= \omega_1 + K \sin(\theta_2 - \theta_1) \\ \dot{\theta}_2 &= \omega_2 + K \sin(\theta_1 - \theta_2), \text{ with } K \geq 0. \end{aligned} \quad (9)$$



In Equation 9,  $\omega_1$  and  $\omega_2$  are the natural frequencies of the two oscillators and  $K$  is a nonnegative coupling constant. Strogatz (1994) presented the more general case with asymmetric coupling between the oscillators, and Yuasa and Ito (1990) described the coordination of arbitrarily many oscillators with more flexible forms of coupling. This system is two-dimensional with each oscillator's instantaneous frequency dependent on both its own natural frequency and the phase difference with its companion. But the model simplifies considerably if we add another level of interpretation. In particular, if we ignore the individual phases,  $\theta_1$  and  $\theta_2$ , and focus instead on the phase difference,  $\phi = \theta_1 - \theta_2$ , then we can convert Equ. 9 to a new dynamical system that models the coordination between the two oscillators:

$$\dot{\phi} = \omega_1 - \omega_2 - 2K \sin(\phi). \quad (10)$$

For the case  $\omega_1 = \omega_2$ , Equ. 10 is essentially the same model we examined previously for the overdamped mass attached to a nonlinear spring (Equ. 5). As before, fixed points for this system occur wherever the velocity of the flow is zero. That is, wherever:

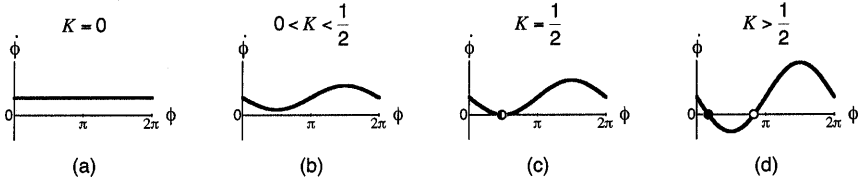
$$\dot{\phi}(\phi)|_{\phi=\phi^*} = 0. \quad (11)$$

The notation in Equ. 11 emphasizes that the rate of flow is a function of the phase difference and that this function evaluates to zero only if  $\phi$  takes on the value of a fixed point,  $\phi^*$ . For the phase difference dynamics, this relationship holds for  $\phi^*$  when  $\omega_1 - \omega_2 - 2K \sin(\phi^*) = 0$ . That is, when:

$$\sin \phi^* = \frac{\omega_1 - \omega_2}{2K} \quad (12)$$

The range of the sine function is bounded on the closed interval  $[-1, +1]$ , and so no fixed points exist, i.e., Equ. 12 has no solutions whenever  $|\omega_1 - \omega_2| > 2K$ . Intuitively, if the natural frequencies are too dissimilar or the nonlinear coupling is too weak, then the oscillators will fail to be synchronized with a constant phase difference (i.e., they fail to become phase-locked or entrained). If  $|\omega_1 - \omega_2| < 2K$ , then two fixed points exist, one stable and one unstable, for every interval of length  $2\pi$  in the domain. In this case synchronization is possible and, for any initial condition not equal to a fixed point value, the nearest unstable fixed point will act as a repeller that pushes the imaginary particle in state space toward one of the neighboring stable fixed points, the attractors. To which attractor the particle is attracted depends on whether the initial condition falls to the left or to the right of the repeller, which acts as the boundary between the basins of attraction for the two stable fixed points in question (see Figure 2).

Now suppose we manipulate the coupling between the oscillators. In particular, let's examine the effects of increasing the value of  $K$  from 0, and, for convenience, we also fix the natural frequencies at  $\omega_1 = 2$  and  $\omega_2 = 1$  (see Figure 3). With no coupling, i.e., when  $K = 0$ , the oscillators behave as independent dynamical systems with one CPG running at twice the rate of the other (Figure 3a). For  $K < 0.5$ , no fixed points exist (Figure 3b) and the coupling is too weak for synchronization, yet the oscillators still respond in a relative coordination mode (Von Holst, 1939/1973). The oscillators never achieve entrainment but instead slow down or speed up, respectively, whenever they get ahead of or behind their companion. Moreover, this effect becomes accentuated as  $K$  increases. Once the coupling constant grows beyond 0.5 (Figure 3d), two fixed points exist (on each  $2\pi$  interval) and entrainment or absolute coordination becomes possible. But what happens at precisely  $K = 0.5$ ? This parameter value is called a bifurcation point because it acts



**Figure 3** — Effects of coupling on entrainment of the coupled oscillator system modeled by **Eqn. 10**. A bifurcation occurs at  $K = 1/2$ , and the resulting fixed point, called a saddle point, has both stable and unstable characteristics.

as a crossroads between two qualitatively different behaviors (see Figure 3c). The theory and taxonomy of bifurcations is beyond the scope of this paper, but one issue we will revisit later is that a bifurcation need not be characterized as a single point with an abrupt change in behavior (e.g., from a walk to a run). The stability of a walking gait, for instance, may drop smoothly as speed increases, and some source of variability may trigger the transition to a running gait well in advance of the speed at which a theoretical bifurcation exists.

In addition to the role that coupling plays for synchronization, the above analysis demonstrates how a distributed control system can coordinate its individual components through limited communication among the components. For example, some models of walking utilize multiple central pattern generators, possibly in a hierarchy, which manage the coordination of degrees of freedom both within and between legs. Variants of the model described here, such as Ermentrout's pulse-coupled oscillator model (Ermentrout & Rinzel, 1984), also explain the way an organism can synchronize with environmental events, such as heelstrike and unanticipated speed changes (Ito et al., 1998; Whelan 1996). In any case, models of coupled oscillators illustrate one way that fixed points are a part of ongoing system behavior and thus one way that discrete and rhythmic movements are related.

### *Poincaré Maps*

French mathematician Jules Henri Poincaré (1854–1912) was a pioneer in the field of topology and is often credited as the founder of the qualitative theory of nonlinear dynamics (Stewart, 1989). Poincaré recognized that all periodic behavior is the same from a topology perspective. Periodic behavior repeats itself, like a circular path, and covers the same ground over and over again in state space. Poincaré's idea, then, was to imagine a plane, now called a Poincaré section, that cuts through state space and to study the way a dynamical system's trajectory pierces that plane. If the system is truly periodic, then the trajectory will intersect the Poincaré section at the same point with each circuit through state space. The utility of Poincaré sections is somewhat more general than one might conclude from our emphasis on strictly periodic behavior. In what follows, we use the periodic case as a means for developing the basic intuition that carries over to higher dimensions and more complicated behavior (e.g., chaotic dynamics).

The practical advantage of a Poincaré section is that it provides a means for constructing a new dynamical system, a Poincaré map, which simplifies the analysis of the original system. We will describe one application of Poincaré maps to gait analysis shortly. More generally, a map is a discrete-time dynamical system

with time jumping from one “tick” to the next rather than progressing smoothly from one instant to another as with a flow. In fact there may be no simple correspondence between discrete “time” and the time that we perceive while watching an analog clock. Each tick could mark the start of a new stage in some process with a variable amount of real time between stages. Nevertheless, concepts such as fixed points and stability apply to maps as well as flows.

For example, consider the following difference equation which represents a simple dynamical system that iteratively scales the current state,  $x_n$ , by a factor  $c$ :

$$x_{n+1} = cx_n. \quad (13)$$

When  $|c| < 1$ , the state shrinks steadily toward  $x = 0$  regardless of the initial state. Conversely, when  $|c| > 1$ , the state forever grows in magnitude unless the initial condition happens to be zero, in which case the state remains at zero until even the slightest perturbation comes along. Thus,  $x = 0$  is a fixed point for this trivial system and  $c$  determines its stability. If we plot Equ. 13 with  $x_n$  as the abscissa and  $x_{n+1}$  as the ordinate, then the dynamic for this map is simply a line and, as with flows, stability is determined by the slope at the fixed point. With maps, however, we examine slope magnitude referenced to unity rather than slope sign referenced to zero (see Kaplan & Glass, 1995; and Strogatz, 1994, for a more detailed review of map stability). One could use a Taylor expansion again to show the link between slope and stability, but the difference here between a map and a flow stems from the way we interpret the dynamic, i.e., the rule that governs system evolution. With maps the dynamic is a transformation from one distinct state to the next, but with flows the dynamic is a description of the infinitesimal change in state over an infinitely short period of time.

Whereas linear maps can at most produce exponential growth/decay or periodic cycles, nonlinear maps can also show quasiperiodic and chaotic behavior. A dynamical systems model in which qualitatively different patterns emerge under systematic scaling of a control parameter,  $r$ , is the classic logistic difference equation by May (1976):

$$x_{n+1} = rx_n(1 - x_n). \quad \{0 \leq r \leq 4\} \quad (14)$$

Complex patterns in these nonlinear systems can be more easily revealed through the application of Poincaré maps. These maps can reveal structure not easily identified in the original state space, help identify the existence of attractors, and reveal the self-similarity present in fractal systems.

Returning to biomechanics, suppose that human walking can be modeled as some dynamical system of the form  $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$ , where  $\mathbf{x}$  is a vector of state variables (e.g., joint positions and velocities) and  $\mathbf{f}$  is a nonlinear function. This model is much too general to be useful, but we could create more insight by deriving the equations of motion based on standard assumptions about rigid bodies and simple revolute joints. We could also use experimental data to construct a numerical approximation of  $\mathbf{f}$ . But if our interest is stability and variability of gait, then Poincaré maps offer a clever alternative. Rather than attempt an analysis of  $\mathbf{f}$ , our next step then is to choose a Poincaré section that cuts through the relevant part of state space. Hurmuzlu et al. (1994) examined several possibilities that correspond to key events during the gait cycle, such as heelstrike and toe-off. The section itself can be defined by setting one of the state variables, such as vertical velocity of the ankle, to a constant value.

A Poincaré section can also be defined by setting a linear combination of state variables to a constant value. Theoretically, almost any section will do, but empirically some choices may be better than others. For instance, state variables related to internal rotation at the hip are a poor choice due to the small variations relative to scale of the overall movement. Once we choose a Poincaré section, we can then collect the needed data. In particular, we record the values of the state variables at the moment the trajectory first intersects the section (e.g., at the instant of heelstrike). Each set of values,  $\mathbf{x}_{n+1}$ , is an iterate of a map,  $\mathbf{P}$ , governed by both the original dynamic  $\mathbf{f}$  and the chosen Poincaré section:

$$\mathbf{x}_{n+1} = \mathbf{P}(\mathbf{x}_n) \quad (15)$$

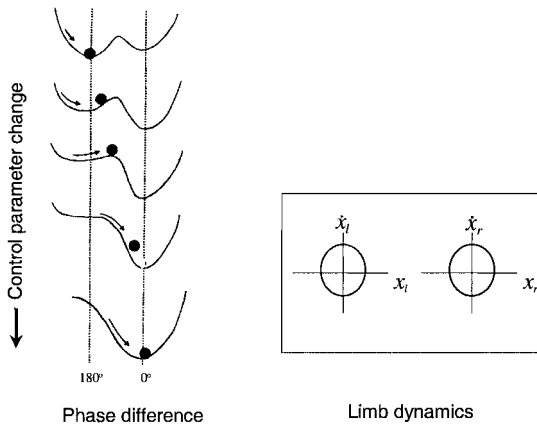
But what have we really gained with this approach? Clearly we replaced a flow with a map, and in the process we decreased by one the dimension of the system we now need to analyze. We also threw away a great deal of data in between heelstrikes (see Dingwell et al., 2001, for an alternative approach that retains the entire data set). The real advantage of this technique is that we replaced a global dynamic  $\mathbf{f}$  with a local dynamic  $\mathbf{P}$ . Then, regression techniques can be used to construct an approximation of  $\mathbf{P}$  and local linearization can be used to quantify the stability of a putative fixed point of  $\mathbf{P}$ . Both Hurmuzlu et al. (1994) and Cheng and Lin (1996) provide further detail about how to apply these techniques to gait analysis. At any rate, stability of the entire gait cycle is linked to the stability of discrete events, like heelstrike. And this allows us to bring to bear many of the same concepts normally attributed to fixed points.

## Stability and Change in Human Movement

### *Pattern Change in Movement Coordination*

A major impetus to understanding the dynamics of coordination change was provided through the work of Kelso (1995). In Kelso's original experiment, participants were asked to oscillate both index fingers in antiphase mode. When individuals systematically increase the oscillatory frequency of both fingers, a sudden and abrupt transition to an in-phase coordination pattern occurs (see Figure 4). An important feature of this transition process is the increase in variability of the state variable (relative phase between the fingers) before the transition. Remarkably, when individuals subsequently reduce their frequency of oscillation, no return to the antiphase mode occurs. Haken et al. (1985) interpreted these transitions as being the hallmark of a self-organizing system in which transitions occur due to the dynamical properties within the system and not from an externally dictated, a priori specified set of instructions. The control parameter in the current paradigm merely enables or facilitates these self-organizing processes to take place.

In the experimental paradigm described above, there are two stable coordinative dynamics, namely in-phase and anti-phase. An essential concept related to the attractors is the basin of attraction (see Figure 2). For a given attractor, the basin of attraction refers to the region in state space from which all initial conditions converge onto the attractor. The depth of the potential well in Figure 4 is an indication of the strength (or stability) of the attractor. When several attractors with different basins of attraction co-exist, a situation is established in which there is multi-stability (Kelso, 1995). Multi-stability occurs when different stable states



**Figure 4 — Left panel: Control parameter change leading to changes in pattern dynamics from antiphase (180°) to in-phase (0°) coupling. These dynamics are fixed-point attractors at either 0 or 180° relative phase. Right panel: Coordination dynamics at the level of the individual limbs are expressed in terms of limit-cycle oscillations. The nature of these limb dynamics could change depending on task and frequency of oscillation.**

emerge for the same value of the control parameter. Bifurcations or phase transitions emerge when, for critical values of the control parameter, the layout or the stability of the attractor changes. Fluctuations play a key role in these qualitative changes in pattern. Kelso and colleagues interpret this transition process as being governed by a switch from a less stable (antiphase) mode to a more stable (in-phase) mode. These multi-stability dynamics were modeled with the following equation:

$$\dot{\phi} = \Delta\omega - a \sin(\phi) - 2b \sin(2\phi) + N. \tag{16}$$

The ratio of  $b/a$  is inversely related to the frequency of oscillation, and  $N$  represents random noise or fluctuations in the system.  $\Delta\omega$  is the difference in natural frequency between the oscillators.

### *The Role of Variability in Pattern Change*

One of the critical elements of the synergetic theory of self-organization lies in the central concept of stability. Phase transitions are characterized by instabilities, which are shown by (a) a strong enhancement of fluctuations (critical fluctuations), and (b) a large increase in the time it takes to recover from a disturbance and return to steady state (critical slowing down).

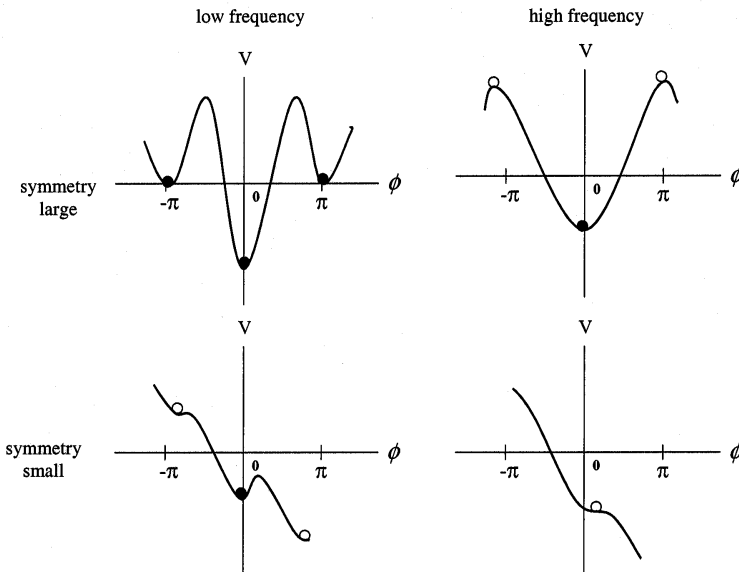
Critical fluctuations in complex systems arise due to influences of elements at a more microscopic level compared to the level of interest. These microscopic influences, together with environmental fluctuations, pull the system away from its current attractor. Fluctuations around the transition point from antiphase to in-phase finger movements are indeed observed in the relative phase patterns (Kelso et al., 1986). Similar fluctuations have been observed in the transition between

walking and running (Diedrich & Warren, 1995) and upper-body phase couplings in the human walking mode (Van Emmerik & Wagenaar, 1996; Wagenaar & Van Emmerik, 2000).

Critical slowing down around the transition point is best shown by the changes and deformations in the potential landscape around the antiphase attractor in Figure 4. As the control parameter reaches its critical point, the potential well becomes more and more shallow and a disturbance away from the fixed point will result in a slow relaxation back to the potential minimum (i.e., the convergence will be longer compared to the situation at lower frequencies when the potential well is steep). Empirical evidence for this critical slowing down in bimanual coordination has been provided by Scholz et al. (1987).

### *Oscillator Asymmetry and Relative Coordination*

In Equation 16 the term  $\Delta\omega$  represents the natural frequency differences between the two oscillators. The transition dynamics presented so far apply to the symmetrical case in which the natural frequencies of the two oscillators are equal. In this original symmetric-coordination example, only the antiphase mode becomes unstable and is annihilated. However, with greater differences in eigenfrequencies between the oscillators, increasing the oscillatory frequency can also result in the disappearance of the in-phase attractor (see Figure 5). Under relatively large asymmetries the two original equilibrium points eventually merge together and even the in-phase attractor becomes unstable at higher frequencies. This is called a saddle-node bifurcation. However, remnants of the original fixed point remain, resulting



**Figure 5 — Potential landscapes of stable and unstable pattern dynamics from the work emerging from the Haken, Kelso, and Bunz (HKB) model of rhythmic coordination. Filled circles: stable states. Open circles: unstable states. Figure adapted from Kelso (1995).**

in an attraction to a certain phase although there is no longer any frequency and phase locking. This loss of stable attractor points with remaining tendencies toward certain phase relations was earlier shown for changes in coupling strength in Figure 3. To reiterate, stable solutions in the form of attractors are formed when the function crosses the ordinate ( $d\phi/dt = 0$ ) and the slope is negative (see also Figure 2). These remnants of fixed points under asymmetrical coupling conditions could be the basis of the phenomenon of relative coordination (Kelso, 1995). This virtual or hidden attractor could be responsible for the observation in relative coordination of a wide range of phase values but still exhibiting a tendency toward one particular phase relation.

The phenomenon of phase fluctuations with a tendency toward phase attraction has also been labeled intermittency. One critical difference between the fluctuations due to intermittency and those due to phase transitions described earlier is that the latter are induced through systematic changes in control parameters. No such external influences are needed for the intermittency mechanism to occur. Instead, under intermittency conditions the system is poised near a critical point where it can spontaneously switch in and out of different states without one state being stable (Kelso, 1995). One interesting consequence of the intermittency mechanism is the insight that, under relative coordination conditions, biological systems tend to “live” near boundaries separating regular and irregular behavior, surviving best near margins of instability. This strategy would afford the best combination of stability and flexibility in behavioral patterns (Beek, 1989; Kelso, 1995). The importance of boundaries or basins between different postural dynamics has recently been emphasized in studies on upright posture control and balance (Ricci, 1993; Van Emmerik & Van Wegen, 2000; Van Wegen et al., 2002).

## Stability and Change in Human Locomotion

### *Assessing Coordination*

Dynamical systems perspectives and analysis tools are gradually gaining prominence in biomechanical research on human locomotion (Burgess-Limerick et al., 1993; Hamill et al., 1999; 2000; Van Emmerik & Wagenaar, 1996). The main contribution is emerging in the assessment and analysis of stability and transitions in coordination patterns during gait. There are various methods for assessing these aspects of coordination (see Hamill et al., 2000, for a review of these techniques).

The main distinction in the literature currently is between continuous and discrete methods of assessing the phase coupling between various segments involved in locomotion. Continuous relative phase (CRP) offers an assessment of the changes in segmental coupling throughout the entire gait cycle. The CRP is derived from the differences in phase angles of two segmental or joint motions. Each phase angle in turn is typically obtained from the position and velocity changes in the phase plane. The calculation of CRP requires various normalization techniques (Hamill et al., 2000; Peters et al., 2003) to properly address issues of coordination and to control for confounding effects of amplitude and frequency of different gait cycles. The calculation of CRP, however, is based on a number of assumptions which, when violated, could affect the outcome of the assessed phase relations. One assumption is that the underlying oscillatory patterns are sinusoidal in nature. Deviations from sinusoidal patterns could introduce dynamics in the

phase relations that are artificial (see Fuchs & Kelso, 1994; Peters et al., 2003; Rosenblum & Kurths, 1998). Moreover, existing relative phase techniques do not apply to conditions in which oscillators are coupled at frequency relations other than one-to-one (multi-frequency couplings).

Another method for assessing coupling changes during locomotion is through the calculation of discrete relative phase couplings. Discrete relative phase (DRP) techniques do not suffer from the assumptions made in the CRP techniques described above. In the discrete relative phase, relevant events in the gait cycle are chosen for comparison such as maximum joint flexion/extension angles during stance, maximum internal/external rotation, etc. DRP is the phase shift in occurrences of these particular events as a function of the cycle period of one of the oscillators. A potential problem with this method is that one must be sure that the coordination of the chosen events is relevant given the task constraints or questions posed. This is important because information from other sections of the gait cycle is not considered.

### *Continuous Relative Phase as a Measure of Coordination*

In research on the influence of movement speed on coordination between pelvic and thoracic rotation, Wagenaar and Beek (1992) observed systematic changes in the phase relation between these segments in the transverse plane. This phase relation changed in healthy persons from a more in-phase pattern at low speeds (i.e., pelvis and thorax move simultaneously and in the same direction) to a more out-of-phase pattern at higher walking speeds (i.e., pelvis and thorax move in opposite directions). It was hypothesized that this change of coordination is necessary for maintaining stability at higher walking velocities.

Applying continuous and discrete relative phase techniques to the coupling of the pelvis and thorax, Van Emmerik and Wagenaar (1996) observed that the variability of the coordination between pelvis and thorax, as measured by the between-cycle variation in their coupling, changed systematically with velocity in young healthy participants. At lower and higher walking speeds, variability was relatively low, with increases at intermediate speeds. The application of continuous relative phase techniques to the coordination of the upper body also revealed a significant lack of adaptation in relative phase between pelvis and thorax in individuals with advanced (Wagenaar & Van Emmerik, 1994) and recently diagnosed Parkinson's disease (Van Emmerik et al., 1999). This reduced ability to change the relative motion between pelvis and thorax from more in-phase to out-of-phase was accompanied by a reduced variability in this coordination pattern. Since variability is a hallmark of phase transitions, it was postulated that this reduced variability is related to the inability in Parkinson's disease patients to make transitions in movement coordination. It was concluded that systematic manipulation of walking velocity can identify coordination deficits in the upper body and that the relative phase analysis can be a sensitive measure for early detection and assessing the effects of therapeutic intervention.

We have also used the continuous relative phase for the assessment of lower extremity coupling and asymmetries during locomotion (Haddad et al., 2001; Hamill et al., 2000). A particularly interesting avenue of research is the link between variability in lower extremity segment and/or joint couplings and the emergence of overuse injuries. Haddad (2000) manipulated asymmetry between coupled oscillators in locomotion through a paradigm of unilateral distal leg loading. He ob-



served that the CRP techniques were much more sensitive in detecting adaptations to imposed asymmetries than traditional joint or segmental analyses. The CRP analysis also demonstrated that the major adaptations to the imposed asymmetries were in the interlimb coupling patterns. The intralimb patterns, in contrast, remained largely invariant under the imposed manipulations (Haddad et al., 2001). Moreover, the CRP analysis revealed systematic adaptations in interlimb coordination in the proximal segments. These adaptations were not observed in the segmental and joint angle displacement data of the proximal joints and segments.

Although the research discussed so far suggests that CRP analyses have been very instrumental in revealing patterns of coordination change and stability under a variety of conditions and populations, several assumptions go into this analysis that might not always be met. In our previous paper, phase angles were calculated from the coordinates  $(\phi, \dot{\phi})$  for each joint angle involved in the assessment of coordination (Hamill et al., 1999). These phase angles typically require normalization to control for differences in the magnitude of the position and velocity signals as well as frequency differences between oscillators (Hamill et al., 2000). Artifacts in this type of phase angle calculation can result when determining relative phase from position-velocity plots (Fuchs & Kelso, 1994; Peters et al., 2003). Recently, the Hilbert transform has been used to address such problems (Rosenblum & Kurths, 1998). In the Hilbert transform all frequency components of the original time series are phase shifted by  $\pi/2$  radians. The resulting signal contains both real and imaginary components, where the real component is the original signal and the imaginary component is the inverse FFT of the phase-shifted frequencies. In lieu of the position-velocity state space, CRP is calculated from the analytic state space as follows:

$$\varphi_1(t) - \varphi_2(t) = \tan^{-1} \frac{\tilde{s}_1(t)s_2(t) - s_1(t)\tilde{s}_2(t)}{s_1(t)s_2(t) - \tilde{s}_1(t)\tilde{s}_2(t)} \quad (17)$$

where  $\tilde{s}_1(t)$  and  $\tilde{s}_2(t)$  are the imaginary components of the Hilbert transform of the two oscillators and  $s_1(t)$  and  $s_2(t)$  are the original joint angles. The variability of the CRP is then assessed over a number of cycles.

Finally, Lamothe et al. (2002) moved away from relative phase techniques as described above and used the difference in the Fourier phase to assess coordination and variability in coordination patterns. This method focuses on the main frequency of one of the oscillators involved and assesses the phasing at that frequency. This method addresses the possible artifacts that may be present in the relative phase analysis due to the fact that the oscillators may not be sinusoidal and phase-locked. However, a drawback of this method is that it only focuses on the main frequency of oscillation, whereby other frequencies and their impact on coordination are not taken into account. In the following section we present a method to assess multifrequency coordination using discrete relative phase and return maps.

### *Multifrequency Coordination: Discrete Relative Phase and Poincaré Maps*

Forms of entrainment at frequency ratios other than 1:1 are possible and have been found in human two-limb coordination (Kelso & Jeka, 1992; Sternad et al., 1999). A dynamical model that has been instrumental in revealing these multifrequency system characteristics is the phase-attractive sine-circle map:

$$\phi_{n+1} = \phi_n + \Omega - K / 2\pi \{1 + A \cos(2\pi\phi_n)\} \sin(2\phi_n) \quad (18)$$

where  $\Omega$  is the frequency difference between the oscillators,  $K$  is the coupling strength, and  $A$  is a measure of relative stability of intrinsically stable states. Changing the oscillator asymmetry and coupling strength allows for an identification of stable coordination with mode locked states, as well as regions in which no stable coordination emerges (Kelso, 1995). In theory, transitions occur between multifrequency states by crossing from one mode locked state to another. The ease of this transition will depend on how deep into a certain mode-locked region the system is. Areas between mode locked regions represent quasiperiodicity with irrational frequency ratios in which no fixed frequency relations exist. The boundaries between mode locked regions might provide the optimal combination between flexibility and stability in movement coordination (Beek, 1989; Kelso, 1995).

Our own research on human locomotor-respiratory coordination focuses on the development of nonlinear dynamical techniques to assess stability and change in multifrequency coupling patterns during locomotion (McDermott et al., 1999; 2003). The coordination between limb movement rhythms and breathing rhythms has been studied in animals and humans (Bramble & Carrier, 1983). In animal gaits, such as those of quadrupeds, a very tight coordination between limb rhythms and breathing rhythms is exhibited due to mechanical constraints in the thoracic region. Horses, for example, have an almost fixed breathing-to-stride ratio of 1:1. This ratio has been found to be due to the constraints put on the thoracic region from the repeated impact loading from the forelimbs striking the ground and the visceral mass dynamics (Baudinette et al., 1987; Bramble & Carrier, 1983; Young et al., 1992). Mechanical constraints during locomotion do not always cause entrainment of the respiratory rhythm but sometimes disrupt it. Lizards, for example, breathe by contracting the intercostal muscles on both sides of the body at the same time. Their locomotion, however, is accomplished by means of a lateral undulatory gait pattern where the intercostal muscles of either side of the body must contract in an alternating fashion. This conflict is described as a speed-dependent axial constraint that results in inadequate lung ventilation during locomotion (Owerkowicz et al., 1999).

Human locomotion has been shown to use a wide range of frequency couplings between movement rhythms and breathing rhythms. These couplings of limb movements to breaths include 1:1, 2:1, 3:1, 3:2, 4:1, and 5:2, with 2:1 being used most often (Bramble & Carrier, 1983; McDermott et al., 2003). In addition, these relationships have been shown to change during running and walking. Bramble and Carrier (1983) reported that experienced runners couple the onset of inspiration and expiration with the airborne phase of the stride cycle when neither foot is contacting the ground. This phase coupling has been shown to change under uphill and downhill locomotory conditions (Takano, 1995). Besides the variety of frequency couplings observed and the changing phase relationships, an additional challenge in assessing these coordination patterns is that they are often highly variable within an experimental trial.

As indicated above, our own research on human locomotory-respiratory coordination focuses on the development of techniques to assess stability and change in these multifrequency coupling patterns. We have examined the effects of locomotory mode and speed, as well as the impact of running experience on these patterns (McDermott et al., 2003). In this study, frequency and phase coupling

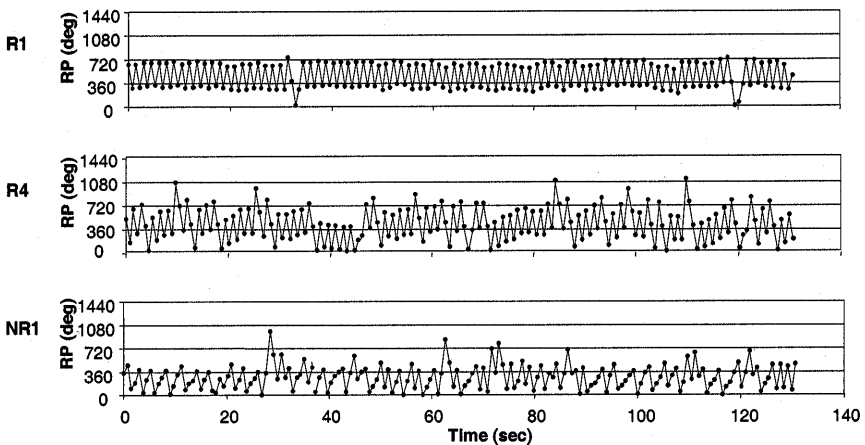
between the end-inspiration (EI) of the breathing rhythm and heelstrike of the locomotor rhythm were obtained from the time series of relative phase:

$$RP = \frac{t+nT}{T} * 360^\circ \quad (19)$$

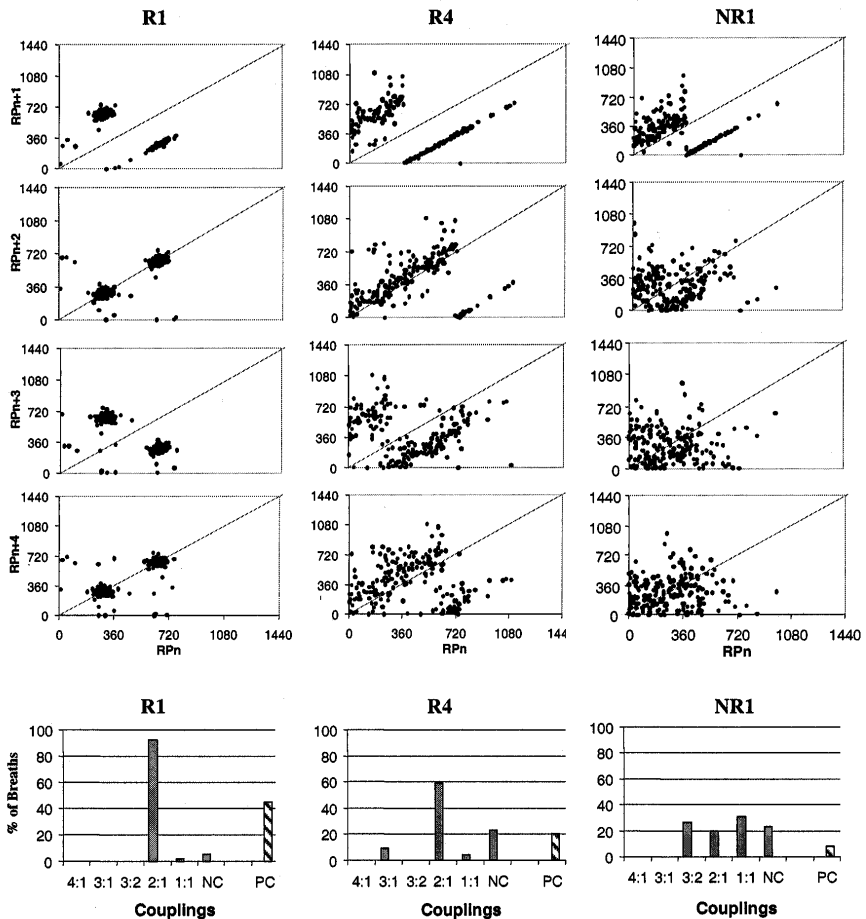
where  $n$  is the number of complete stride cycles between each heelstrike and the subsequent EI,  $T$  is the time duration of the stride in which EI occurred, and  $t$  is the time lag from the beginning of the stride in which EI occurred to the subsequent EI. Return maps and systematic time lags were used to illustrate different frequency and phase couplings (see also Kelso & Jeka, 1992). This technique is beneficial for the study of locomotory and breathing rhythm coordination because it identifies consistently occurring coupling patterns without setting criteria a priori.

Next, a distribution of frequency couplings and noncouplings was calculated for each trial, expressing the occurrence of coupling as a percentage of the total number of breaths. Since it has been reported that multiple couplings are commonly observed during an experimental period, the dominant coupling was considered to be the one that occurred most often. The percentage of breaths in which the dominant coupling occurred was defined as the strength of coupling. Variability in frequency coupling was measured by the contributions of the second dominant coupling (SC) as well as noncouplings (NC).

To illustrate the process of frequency coupling assessment, example relative phase time series are plotted in Figure 6 for three participants running on a treadmill at 20% above the preferred running speed. To demonstrate the periodicity of these time series, return maps of the data are plotted in Figure 7 (top panel) using Lags 1–4. Frequency couplings were identified in these maps by applying specific range criteria to the appropriate map (see McDermott et al., 2003, for a detailed



**Figure 6** — Time series of relative phase between heelstrike and breathing (end-inspiration) for two runners (R1, R4) and a nonrunner (NR1) obtained during running at 20% greater than preferred running speed. Consistency of the relative phase pattern for R1 indicates absolute coordination at a 2:1 frequency ratio. R4 shows less consistency but still some tendency toward certain phase relations also utilizing a 2:1 ratio (relative coordination). NR1 on the other hand shows little tendency to a specific phase and frequency pattern.



**Figure 7 — Top:** Example return maps for the relative phase data presented in Figure 6. For each map, each relative phase value ( $RP_n$ ) is plotted vs. the relative phase value with the appropriate time lag ( $RP_{n+1}$ ,  $RP_{n+2}$ ,  $RP_{n+3}$ ,  $RP_{n+4}$ ). **Bottom:** Solid bars: Percentage of breaths occurring at each frequency coupling for the three example data series. Hatched bars: Strength of phase coupling.

overview of the methods). Points identified by these criteria in the maps represent the phase relation of heelstrikes occurring during consecutive breath cycles. From this, the extent to which each frequency coupling occurred in the series was calculated and is represented in Figure 7 (bottom panel).

This process quantifies the varying frequency coupling patterns that are observed both within and across the example data sets *independent* of the phase relationships that occur. Since phase is often dependent on the frequency relationship, particularly for half-integer couplings, the next step was to quantify the phase coupling based on the frequency couplings identified in each time series. To accomplish this, phase coupling (PC) was assessed by the dispersion of points

(noncouplings not included) from the line of identity in the lowest range of each return map (between 0 and 360). Perfect phase coupling (no variability) would be evident if all points lay on the line of identity and variability in phase coupling as deviations from this line. Therefore, phase coupling (PC) was quantified by first calculating the Euclidian distance of each point ( $d_n$ ) from the line of identity and then summing the weighted distances ( $wd$ ):

$$wd_n = \begin{cases} 1 - \frac{|d_n|}{40 * \cos(45)}, & d_n \leq 40 \\ 0, & d_n > 40 \end{cases} \quad (20)$$

$$PC = \frac{\sum_{n=1}^m wd_n}{m} * 100 \quad (21)$$

where  $m$  is the number of points in the lowest range of each return map. This measure essentially weights points with distances from the line of identity that are greater than or equal to  $40^\circ$  by 0, and those with distances less than  $40^\circ$  according to their distance, and expresses their sum as a percentage of highest possible sum. Perfect phase coupling would therefore result in a  $PC = 100\%$  of breaths and the amount of variability in the phase coupling is proportional to the decrements from 100%. The resulting PC values for the example data are also plotted in Figure 7 (bottom panel). These results quantitatively show the different frequency couplings utilized in these data sets as well as the overall strength of the phase coupling. We have used this methodology over a range of locomotory speeds in order to determine the influence of running training on locomotor-respiratory coordination (McDermott et al., 2003). The focus was not only on the coordination patterns within each condition but also how the patterns change across conditions.

The results showed that runners differ from nonrunners not in the strength of the coupling per se, but how the frequency coupling responds to varying speed; the runners showed *greater stability* in the frequency coupling across locomotory speeds. The differences in stability, in turn, reflect different adaptive strategies of the respiratory and/or stride rhythms to the changing mechanical and physiological demands of the locomotory speeds and may have implications for training and rehabilitation programs where movement efficiency is an important factor.

The use of discrete relative phase in conjunction with systematic time lags and return maps has allowed both frequency and phase coordination patterns to be quantified in a system that is highly variable and where multiple frequency couplings are present. These patterns could then be assessed in light of changes in a relevant parameter such as locomotory speed to reveal different transition dynamics. The utility of this analysis in this sense suggests it can be useful in helping us understand complex coupled biological oscillator systems and changes that occur in these systems as a result of training, disease, and aging.

## Summary and Conclusions

Nonlinear dynamics tools and techniques are gradually becoming more firmly embedded into biomechanics and human movement research. It is hoped that the modeling techniques described in this paper will provide the reader with enough

information and insight to delve deeper into the richness that nonlinear and complex systems approaches have offered the scientific community. Indeed, at all levels of analysis, ranging from molecular to whole organ or system levels, complex systems and nonlinear dynamics techniques and analysis procedures are being used.

The Bernstein “revolution” has resulted in a significantly different outlook on movement control over the last few decades. From a focus on relatively simple movements emerged a research enterprise that has as its primary goals to study movement in the context in which the complexity of the patterns are allowed to emerge. The approach taken is that the control of systems with many degrees of freedom is not necessarily more difficult or complex than that of systems only comprising a few degrees of freedom. What is acknowledged is that complex patterns might not require complex control structures. The original work of Robert May (1976) in the area of population dynamics has been instrumental in showing the emergence of complex patterns under systematic scaling of a single control parameter.

The presented overview on the mathematical underpinnings of nonlinear dynamics and some of its basic analysis tools should provide the reader with a sufficient understanding to delve deeper into the issues raised in this paper (e.g., see Strogatz, 1994). Developments in nonlinear science over the past three decades have brought into the human movement domain a sophisticated and broad set of principles and analysis tools for studying stability and change. Regardless of focus or paradigmatic orientation, a better and more complete understanding of the nature of pattern change is fundamental for research in motor development, motor learning, and movement disorders.

In the final segment of this paper, we discussed recent progress in the application of nonlinear dynamics techniques to the study of human locomotion. In particular, we assessed the contribution of continuous and discrete relative phase techniques to the understanding of stability and change in human locomotion. This area of study brings many complexities with it, as an understanding of gait cannot simply emerge from the coordination patterns and couplings within and between limbs in the lower extremity. An important focus for future development is the assessment of upper extremity, trunk, and head coordination and their contribution to instability during locomotion in a variety of disorders. The techniques and principles derived from nonlinear dynamical approaches offer excellent and exciting prospects for the biomechanical study of human locomotion.

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