The Evolution and Ontogeny of Ordinal Numerical Ability
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It is generally assumed that the development of human mathematical reasoning requires years of schooling. That being the case, mathematical reasoning would seem beyond the reach of the rest of the animal kingdom. This common-sensical conclusion poses an issue that is the focus of this chapter. What, if any, evolutionary precursors of human mathematical reasoning can be observed in animals?

To answer that question, we must first recognize that human mathematical ability is composed of many heterogeneous skills. Humans use symbols to represent numerosities and to represent operations such as addition and division and are capable of manipulating numerical symbols in complicated ways (e.g., algebra and the calculus). It is even more important to recognize that the most basic numerical skills don't require *any* numerical symbols. It is, for example, possible to discriminate the relative numerosity of two sets of objects without the help of numerals (e.g., that a collection of 4 peanuts is larger than a collection of 2 apples).

During the past 30 years, investigators of animal behavior have shown that many species possess some numerical ability (for reviews see Davis and Perusse 1988; Roberts 1997). Those observations have led some psychologists to hypothesize that human mathematical ability evolved from numerical abilities that can be observed in animals (Dehaene 1997; Gallistel and Gelman 1992, 2000). Our research program on the ordinal numerical abilities of rhesus monkeys has provided considerable evidence in support of that hypothesis (Brannon and Terrace 1998, 1999, 2000). As background, we will first describe other experiments that have addressed this topic and show how our approach differs. We will then discuss aspects of a monkey's numerical behavior that appear to be analogs of mathematical thinking in adult and developing humans. Finally, we define some promising future directions for research.

If monkeys use number to organize events in their natural environment we should expect them to represent number on at least an ordinal scale. They should not only be able to differentiate n versus m objects, but they should also appreciate that a collection of $n + m$ objects is numerically greater than n objects. Thomas, Fowlkes and Vickery (1980) tested this idea in an experiment in which squirrel monkeys were trained to respond to the lesser of two numerosities. The values of the numerosities were increased progressively as the monkeys learned each pair. Although Thomas, et al. provided impressive evidence that squirrel monkeys could discriminate sets containing as many as 10 and 11 elements, it was unclear whether their subjects used an ordinal rule to solve each pair, or whether they had simply learned a series of pair-wise discriminations, for example, that 4 is rewarded when it is paired with 5, but not when it is paired with 3, etc. The latter interpretation cannot be ruled out because the pairs of numerosities were trained successively, one pair at a time.

Washburn and Rumbaugh (1991) used a different paradigm to investigate the numerical abilities of rhesus monkeys. On each trial, they presented a pair of Arabic numerals whose values ranged from 1-9. The monkeys learned to choose the larger numeral and even responded correctly when novel combinations of Arabic numerals were tested. Although the monkeys learned to choose the larger numeral when it was presented in a novel pair, it doesn't follow that they learned a symbolic numerical rule. The monkey's choices could have been based on the hedonic value associated with each of the numerals (yummie vs. very yummie, also see Olthof et al. 1997). To show that the monkeys associated a discrete number of pellets with each Arabic numeral, it would be necessary to test them with a paradigm that provided the same amount of food for each correct choice.

Other investigators have used a forced-choice discrimination procedure to study ordinal numerical knowledge (Meck and Church 1983; Roberts and Mitchell 1994; Emmerton et al. 1997). In these studies rats and pigeons were trained to make one response to a small number of stimuli (sounds and/or light flashes) and another response to a larger number. When