

$$\begin{array}{c}
 F \vee T \qquad T \vee F \\
 T & \& T \\
 \hline
 T
 \end{array}$$

$$\begin{array}{c}
 \text{Conclusion} \quad (Raa \& Rba) \vee (Rab \& Rbb) \\
 F \& T \qquad T \& F \\
 F \qquad \vee \qquad F \\
 \hline
 F
 \end{array}$$

**Exercise 4.3.1** Construct countermodels for the following invalid sequents.

- i\*  $\exists x F_{xx} \vdash \forall xy F_{yx}$
- ii\*  $\forall y \exists x F_{xy} \vdash \exists x F_{xx}$
- iii\*  $\forall x \exists y F_{xy} \vdash \exists x \forall y F_{xy}$
- iv\*  $\forall x \exists y \sim F_{xy}, \forall x \forall y (G_{xy} \rightarrow \sim F_{xy}) \vdash \forall x \exists y \sim G_{xy}$
- v\*  $\forall x (F_x \rightarrow \exists y G_{xy}) \vdash \forall x \forall y (F_x \vee \sim G_{xy})$
- vi\*  $\forall x \exists y \forall z V_{xyz} \vdash \exists y \forall x \forall z V_{xyz}$
- vii\*  $\forall x \sim \forall y T_{xy} \vdash \forall x \sim \exists y T_{xy}$
- viii\*  $\exists xyz ((F_{xy} \& F_{yz}) \& \neg(F_{xz} \vee F_{yx})) \vdash \forall x \exists y F_{yx} \rightarrow \forall x \sim F_{xx}$
- ix\*  $\forall x \exists y F_{xy}, \exists x \sim \forall y G_{yx}, \exists xy F_{xy} \leftrightarrow \exists xy (G_{yx} \& \sim G_{xy})$   
 $\qquad \qquad \qquad \vdash \exists y (G_{xy} \vee G_{yx})$
- x\*  $\exists x \exists y F_{xy} \leftrightarrow \sim \exists x G_{xx}, \forall y \exists x G_{yx} \vdash \forall x \sim F_{xx}$