

requires first the replacement of the object language names with their metalanguage equivalents, yielding:

$$\forall x(Fxc \rightarrow x \neq c).$$

Then expansion proceeds by the normal method, to yield:

$$(Fcc \rightarrow c \neq c) \& (Fdc \rightarrow d \neq c).$$

Finally, given the predicate extension

$$F: \{\langle c, d \rangle, \langle d, c \rangle\}$$

the conjunction can be seen to be true given the falsity of the antecedent of the left conditional and the truth of both antecedent and consequent on the right.

- Exercise 4.4** Construct countermodels for the following sequents.
- i* $a=b, c=d \vdash a=c$
 - ii $Fa, a \neq b \vdash \sim Fb$
 - iii $\forall x \exists y x=y \vdash \exists y \forall x y=x$
 - iv $\exists x(x \neq a \rightarrow Fx), a=b \vdash Fb$
 - v* $\exists xy((Fx \& Fy) \& x \neq y) \vdash \forall x Fx$
 - vi $\forall xy(Fx \& Gy \rightarrow x=y) \vdash \sim \exists x(Fx \& Gx)$
 - vii $\exists x(x \neq a \rightarrow Fx \vee Gx) \vdash \exists x(Fx \vee Gx \rightarrow x \neq a)$
 - viii $\forall xy(Fxy \rightarrow y=x) \vdash \exists x Fxx$
 - ix* $\exists x \forall y(Fxy \leftrightarrow x \neq y) \vdash \forall xyz((Fxy \& Fxz) \rightarrow y=z)$
 - x $\forall xy((Fx \& Fy) \rightarrow x \neq y)$
 $\vdash \forall xyz(((Fx \& Fy) \& Fz) \& ((x \neq y \& y \neq z) \& x \neq z))$