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|------|------|---|---------------------------|
| 5 | (5) | $\sim Ha$ | A |
| 5 | (6) | $\exists x \sim Hx$ | 5 $\exists I$ |
| 4 | (7) | Ha | 4,6 RAA (5) |
| 4 | (8) | $Fa \vee Ha$ | 7 $\vee I$ |
| 1,4 | (9) | $Ga \& Ka$ | 3,8 $\rightarrow E$ |
| 1,4 | (10) | $Ka \& Ga$ | 9 & Comm |
| 1,4 | (11) | $\forall x(Kx \& Gx)$ | 10 $\forall I$ |
| 1,2 | (12) | $\exists x \sim Hx$ | 2,11 RAA (4) |
| S107 | | $\forall x(Fx \leftrightarrow \forall yGy) \vdash \forall xFx \vee \forall x \sim Fx$ | |
| 1 | (1) | $\forall x(Fx \leftrightarrow \forall yGy)$ | A |
| 2 | (2) | $\sim \forall xFx$ | A [for $\rightarrow I$] |
| 3 | (3) | $\sim \exists x \sim Fx$ | A [for RAA] |
| 4 | (4) | $\sim Fa$ | A |
| 4 | (5) | $\exists x \sim Fx$ | 4 $\exists I$ |
| 3 | (6) | Fa | 3,5 RAA (4) |
| 3 | (7) | $\forall xFx$ | 6 $\forall I$ |
| 2 | (8) | $\exists x \sim Fx$ | 2,7 RAA (3) |
| 9 | (9) | $\sim Fa$ | A [for $\exists E$ on 8] |
| 1 | (10) | $Fa \leftrightarrow \forall yGy$ | 1 $\forall E$ |
| 1,9 | (11) | $\sim \forall yGy$ | 9,10 BT |
| 1,2 | (12) | $\sim \forall yGy$ | 8,11 $\exists E$ (9) |
| 13 | (13) | $\exists xFx$ | A [for RAA] |
| 14 | (14) | Fa | A [for $\exists E$ on 13] |
| 1,14 | (15) | $\forall yGy$ | 10,14 BP |
| 1,13 | (16) | $\forall yGy$ | 13,15 $\exists E$ (14) |
| 1,2 | (17) | $\sim \exists xFx$ | 12,16 RAA (13) |
| 19 | (18) | Fa | A [for RAA] |
| 19 | (19) | $\exists xFx$ | 18 $\exists I$ |
| 1,2 | (20) | $\sim Fa$ | 17,19 RAA (18) |
| 1,2 | (21) | $\forall x \sim Fx$ | 20 $\forall I$ |
| 1 | (22) | $\sim \forall xFx \rightarrow \forall x \sim Fx$ | 21 $\rightarrow I$ (2) |
| 1 | (23) | $\forall xFx \vee \forall x \sim Fx$ | 22 $\vee \rightarrow$ |
| S109 | | $\forall x(Dx \rightarrow Fx) \vdash \forall z(Dz \rightarrow (\forall y(Fy \rightarrow Gy) \rightarrow Gz))$ | |
| 1 | (1) | $\forall x(Dx \rightarrow Fx)$ | A |
| 2 | (2) | Da | A |